

necessary condition for torsional stability. Thus  $|\epsilon|$  remains small in the case where  $\Lambda$  tends toward zero. In general  $\epsilon$  may be written as

$$\epsilon = [(\epsilon^*)^{1/2}/(3)^{1/4}](\Lambda^{1/2}/\tilde{p}_0)(\tau^*/\tau_{cr}) \quad (12b)$$

In the case where  $\Lambda$  is of order one, then Eq. (7) shows that  $|\Lambda^{1/2}/\tilde{p}_0|$  is of order one. Since for thin-walled shells,  $(\epsilon^*)^{1/2} \ll 1$ , then it follows that, in this case, all  $|\epsilon| \ll 1$ . Therefore, for thin-walled shells and  $n$  moderately large,  $|\epsilon| \ll 1$  may reasonably be assumed for all  $\tau^*$  less than the critical buckling value  $\tau_{cr}$ . Thus  $\epsilon^2$  and higher-order  $\epsilon$  terms may be neglected for the evaluation of  $\alpha$  in Eq. (8).

Let

$$\Lambda/4 \equiv \tilde{p}_0^2 \beta \quad (13)$$

where  $\beta$  is a complex number. Then Eq. (6) can be conveniently rewritten with the help of Eqs. (8, 9, and 13) as

$$\pm i(1 + \alpha\epsilon)^2 \exp(\mp \epsilon/1 + \alpha\epsilon) = \tilde{p}_0^2[(1 + \alpha\epsilon)^2 - \beta]^2 \quad (14)$$

Rearrangement of Eq. (14) as a linear function of  $\epsilon$  gives

$$\left(2\alpha \mp 1 - \frac{4\alpha}{1 - \beta}\right)\epsilon \pm \left[i\frac{\pi}{2} - 2\log\tilde{p}_0 - 2\log(1 - \beta)\right]\epsilon^0 = 0 \quad (15)$$

Since Eq. (15) is assumed to be true for some range of  $0 \leq \epsilon \ll 1$  in which  $\epsilon$  can take any value, then it follows that Eq. (15) can be established only if the coefficients of  $\epsilon$  are identically zero. The coefficient of  $\epsilon^0$  is automatically satisfied, since  $\tilde{p}_0$  satisfies Eq. (6) with  $\rho_1 = 0$ . Both  $\tilde{p}_{01}$  and  $\tilde{p}_{02}$  have been defined so as to take the plus (+) sign in the coefficient of  $\epsilon$  as can be verified by comparing Eq. (6a) with the identity [Eq. (5a) of Ref. 2]

$$[\tilde{p}_{0j}^2 - (\Lambda/4)]^2 = i\tilde{p}_{0j}^2 \quad (j = 1, 2) \quad (16)$$

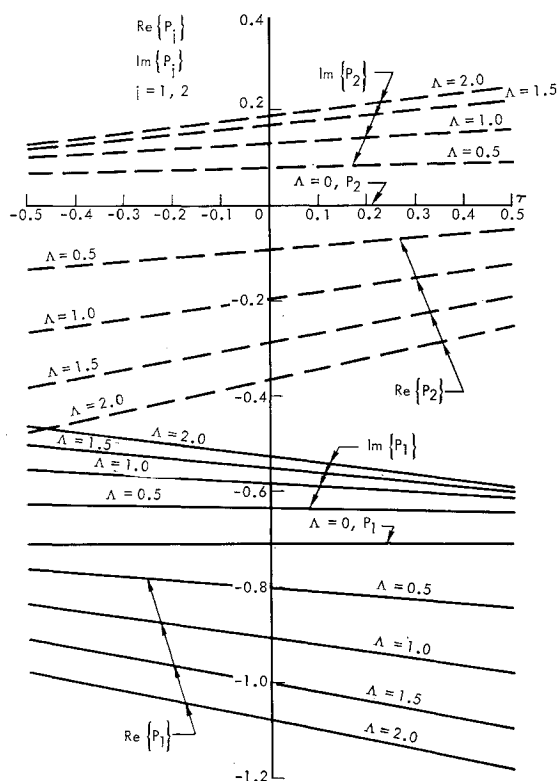


Fig. 1 Approximation to real and imaginary roots of Eq. (4) as function of shear stress ratio  $\tau$  for various values of circumferential lobe number parameter  $\Lambda$ .

Similarly, when  $\rho_1 \neq 0$ ,  $\alpha$  is taken to be

$$\alpha = (\beta - 1)/2(\beta + 1) \quad (17)$$

The combination of Eqs. (8) and (17) yields the required roots, namely,

$$p = \tilde{p}_0\{1 + [(\beta - 1)/2(\beta + 1)]\epsilon\} \quad (18)$$

Substitution of the listed definitions of  $\tau$  and  $\Lambda$  in Eq. (12b) leads to

$$\epsilon = \Lambda\tau/2(3)^{1/4}\tilde{p}_0 \quad (19)$$

Therefore, Eq. (18) can be rewritten by making use of Eq. (19) as

$$p = \tilde{p}_0 + [(\beta - 1)\Lambda/4(3)^{1/4}(\beta + 1)]\tau \quad (20a)$$

$$= \tilde{p}_{0j} + \beta_{0j}\tau \quad (j = 1, 2, 3, 4) \quad (20b)$$

where

$$\beta_{0j} = (\beta - 1)\Lambda/4(3)^{1/4}(\beta + 1) \quad (21a)$$

or

$$\beta_{0j} = (\Lambda - 4\tilde{p}_{0j}^2)\Lambda/4(3)^{1/4}(\Lambda + 4\tilde{p}_{0j}^2) \quad (21b)$$

if Eq. (13) is introduced in Eq. (21a).

The roots of  $p$  vs  $\tau$  are shown in Fig. 1. It is revealed that, for  $\Lambda > 0$  and  $\tau \geq 0$ ,  $p_3 = \tilde{p}_1$  and  $p_4 = \tilde{p}_2$ . The proof is obvious since  $\tau$  is real and the conjugate relation holds in Eq. (20b). Finally, the negatives of these four roots complete the eight distinct roots of  $p_j$  in Eq. (4).

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## Review of Recent Developements in Turbulent Supersonic Base Flow

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## Introduction

TWO-DIMENSIONAL supersonic base-pressure problems and, in particular, the flow over a backward-facing step have been successfully analyzed, to a certain extent, by various authors.<sup>1-5</sup> The flow model used by most authors has four basic components (the mixing region, the region of confluence, the wake, and the main flow). Of these, the reattachment phenomena in the region of confluence is rather an important one and, in the case of flow over a step, dominates the variation of base pressure. Setting the pressure rise to reattachment equal to the difference between the base pressure and the pressure far downstream, Korst<sup>4</sup> obtained

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a solution to the base-flow problem which found considerable experimental verification.<sup>6-8</sup>

Recently, Nash<sup>9</sup> criticized Korst's criterion and suggested that the limiting ( $\theta/t = 0$ , where  $\theta$  is the boundary-layer momentum thickness at separation and  $t$  is the base height or trailing-edge semithickness) base pressure predicted by Korst does not correspond to the minimum observed base pressure. He then proceeded to formulate a modified analysis reported to be more consistent with experimental observations. Nash argues that the fundamental principles of Korst's theory and the details of the solution are not fully supported by observations. For example, 1) tests on transitional base flow, for a range of Reynolds numbers reported by Gadd et al.<sup>10</sup> and van Hise<sup>11</sup> in the absence of suction or other conditions with similar effects, give base pressures substantially lower than the limiting values predicted by Korst; 2) a simple linear extrapolation of existing data at small but finite values of  $\theta/t$  to  $\theta/t = 0$  leads to an overestimation of base pressure; and 3) the variation of base pressure with the thickness of the boundary layer approaching the trailing edge or separation corner cannot be accounted for quantitatively by an extension of Korst's method, and the theory<sup>12,13</sup> predicts higher base pressure for given boundary-layer thickness than is observed experimentally.

The present paper is an attempt to point out, first, the difficulties in using the preceding arguments to discredit Korst's theory, and second, some of the difficulties associated with the theory of Nash and their effect on base pressure.

## Discussion

### 1. Base pressure in flow over a backward-facing step vs that for flow over an isolated airfoil section

Nash contends that the minimum base pressure is obtained when transition occurs just upstream of the separation corner. However, the experimental minimum does not always occur at the transition Reynolds number as the theory<sup>2,9</sup> requires, and it may well be that the reason for the minimum differs from that suggested by the theory. Gadd et al.<sup>10</sup> suggest that the cause might be some variation in the character of the turbulent flow that may occur close to transition. Also, since the recompression shock at reattachment oscillates about the point of reattachment, the pressure in the "dead-air region" varies about a mean value. These oscillations in pressure propagate upstream through the subsonic portions of the free shear and boundary layers, causing the transition point to oscillate about the corner. Thus it is possible that the minimum base pressure is associated with this unsteady phenomena.

The measurements of Gadd et al.<sup>10</sup> were made on models with small span, and three-dimensional effects may have been present, resulting in lower values of base pressures. Therefore, such measurements cannot be used to check the theory. In the experiments of van Hise on ogives and wedges, care was taken to minimize three-dimensional effects. However, base pressures substantially lower than the minimum value predicted by Korst were measured over a range of Reynolds numbers. For example, at a Mach number of 2.2, the ratio of limiting base pressure to ambient static pressure computed from Korst's theory is 0.30, whereas values as low as 0.23 were recorded on ogive models with thickness to chord ratio of  $\frac{1}{8}$ . Since both experiment and theory indicate that base pressure is greatly lowered by negative bleed and suction, the possibility that end effects, however small, were present in the latter tests will lower the base pressure. However, van Hise's data or any other data obtained from tests on blunt-trailing-edge sections cannot be used to check Korst's theory, because the details of the reattachment mechanism for the case of flow over a blunt-trailing-edge section are possibly very different from those for the flow over a backward-facing step (the flow model used in Korst's theory). Among others, one such difference is that in the

flow over the step the separated shear layer reattaches to the downstream wall where a no-slip condition is satisfied, whereas in the flow over an isolated section reattachment takes place along the centerline, where the boundary condition is that the pressure be equal on both sides (a condition that allows for motion on the boundary). The effect of this condition on the base pressure is similar to the effect of applying suction near the reattachment zone in the flow over a step. Therefore, it is reasonable to assume that base pressure on an isolated section, even in the absence of three-dimensional effects, is lower than that on a two-dimensional backward-facing step. Some confirmation of this point is found in the results of base-pressure tests by Chapman et al.<sup>6</sup> on airfoil sections at a Mach number of 2.0 and for a wide range of  $\theta/t$  which fell considerably below the data points obtained by Thomann<sup>14</sup> at  $M = 1.8$  and Badrinarayanan<sup>15</sup> at  $M = 2.07$  in tests on backward-facing steps. Nash suggests that these results indicate that, when the thickness of the boundary layer is large, the variation of base pressure with momentum thickness on a blunt-trailing-edge section differs from that for the backward-facing step. However, in view of the foregoing discussion, we believe, regardless of the magnitude of  $\theta$ , that the flow over a backward-facing step is fundamentally different from the flow over a blunt-trailing-edge section.

### 2. Extrapolating base pressure from small finite values of $\theta/t$ to $\theta/t = 0$

Nash states that, in the neighborhood of  $\theta/t = 0$ , the curve representing the variation of base pressure with boundary-layer thickness at a given Mach number has both a large slope and a large negative second derivative. Thus, a linear extrapolation of the curve from some position of finite  $\theta$  to  $\theta = 0$  could result in serious overestimation of the limiting base pressure.

Recent measurements by Roshko and Thomke<sup>16</sup> of turbulent base pressure on a step on an axisymmetric model (thus avoiding end effects) with ratio of boundary-layer thickness to base height as low as 0.006 at freestream Mach number  $M_\infty = 2.09$  extrapolate linearly to Korst's value. At  $M_\infty = 4$ , the results follow the trend predicted by Nash but lie considerably above his theoretical curve. However, according to Charwat and Yakura,<sup>17</sup> the interaction between the component parts of the model becomes evident at a Mach number of 3, and hence the flow model and the analysis become less valid as the Mach number increases. One may also argue that the "error function" velocity profile used in the two-dimensional theory is not applicable to the axisymmetric case, and hence it is not appropriate to use the data of Ref. 16 in support of the present argument. However, the largest ratio of step height to body radius of 0.28 used in the tests of Ref. 16 is perhaps sufficiently small to minimize any such problems. Furthermore, data of tests<sup>8,14,15</sup> on two-dimensional backward-facing steps, carried out at Mach numbers of about 2 and  $\theta/t$  ratios as low as 0.0125, extrapolate linearly to  $P_b/P_1$  value of 0.29, whereas Korst's value is 0.35, and that predicted by Nash is considerably lower at 0.14. Similar tests carried out by Carriere and Sirieix<sup>12</sup> at Mach number of 3 and for  $\theta/t$  ratios as low as 0.008 also extrapolate linearly to approximately the Korst value of 0.185, whereas Nash predicts the much lower value of 0.105.

Thus, results of a large number of experiments on the effects of boundary-layer momentum thickness on  $P_b/P_1$  at Mach numbers less than or equal to 3 with  $\theta/t$  as low as 0.008 extrapolate linearly to approximately the Korst value, whereas Nash predicts much lower values for the same test conditions.

### 3. Effect of approaching boundary layer on base pressure

Kirk<sup>5</sup> suggests that the effect of the approaching boundary layer on base pressure may be taken into account simply by

replacing the real free shear layer by an "equivalent asymptotic" free shear layer growing over a greater length from zero thickness. The distance between the origin of the equivalent system and the separation point is assumed proportional to the momentum thickness at separation. Consequently, the value of  $\theta$  presented to the mixing layer is very important.

At supersonic speeds, one expects some changes in the velocity profile as the boundary layer interacts with the centered expansion at the corner. In order to take proper account of the boundary-layer effect on base pressure, Nash attempts to compute its characteristics immediately downstream of the expansion fan. He obtains a relation that to a first approximation

$$\rho_2 u_{e2} \theta_2 / \rho_1 u_{e1} \theta_1 = (M_{e1}/M_{e2})^2 \equiv r \quad (1)$$

where  $\rho$  is density,  $u$  velocity, subscripts 1 and 2 refer to conditions upstream and downstream of the expansion fan, and  $e$  refers to conditions at the outer edge of the boundary layer.

In arriving at Eq. (1), Nash assumes that in the boundary layer most of the mass flux passes through stream tubes along which the mean velocity is nearly equal to the velocity in the external stream, "full" profile (well-developed turbulent flow). This assumption, and hence Eq. (1), may result in reasonable estimates of  $\theta_2$  if the profile introduced to the centered expansion is "full." However, certainly this assumption is not valid when the profile is not "full" as when transition occurs just upstream of the base, where the boundary-layer thickness and hence base pressure are at a minimum. In this case,  $z = 1 - u_1/u_{e1}$  is not small, and the expansion of  $u_2/u_{e2}$  [Eq. (3.7) of Ref. 9] in terms of  $z$  and  $r$  must contain higher-order terms. This would, in effect, result in

$$\rho_2 u_{e2} \theta_2 = r \rho_{e1} u_{e1} (\theta_1 - D) \quad (2)$$

where  $D$  is the sum of terms involving  $r$ ,  $r^2$ ,  $\theta_1$ ,  $\delta^*$ ,  $\delta^{**}$ , etc.

Although the interaction length is short and the pressure gradients are much larger than those normally encountered in boundary-layer phenomena, viscous effects are present, and the boundary-layer expansion is a dissipative process. For a dissipative shear flow, one may write for the expansion from  $p_1$ ,  $T_1$  to  $p_b$ ,  $T_2$

$$T_1/T_2 = \eta_1 (T_{e1}/T_{e2}) = \eta_1 (p_1/p_b)^{(\gamma-1)/\gamma} \quad (3)$$

Also, conditions in the boundary layer may be expressed in terms of freestream conditions

$$u^2 + 2C_p T = \eta(u_e^2 + 2C_p T_e) \equiv u_e^2 + 2C_p T_e' \quad (4)$$

where  $\eta_1$  and  $\eta$  are constants less than unity and  $T_e'$  is less than  $T_e$ . Then, along a streamline, conditions at the beginning and end of the interaction are related by

$$(1 - u_e^{*2}/1 - u_1^{*2}) = (u_{e1}^2/u_{e2}^2)(T_2 - T_{e2}'/T_1 - T_{e1}') \equiv r' \quad (5)$$

Thermodynamic considerations of the expansion processes and the relative values of  $T_1$ ,  $T_2$ ,  $T_{e1}$ , and  $T_{e2}$  indicate that  $r' < r$ , where  $u^* = u/u_e$  and

$$r \equiv (u_{e1}^2/u_{e2}^2)(T_2 - T_{e2}/T_1 - T_{e1}) \quad (6)$$

Using this value of  $r'$  in Eq. (2) shows that the actual value of  $\theta_2$  is lower than that predicted by Nash according to Eq. (1). Therefore, since the theory predicts  $P_b/P_1$  to increase with  $\theta_2$  [Eq. (6.2) of Ref. 9], the theoretical variation of  $P_b/P_1$  with momentum thickness should, especially at larger Mach numbers, fall above the experimental data (Fig. 3 of Ref. 9), and the theory would indicate a faster rate of variation with momentum thickness than does the experiment.

Furthermore, the Mach number distribution throughout the boundary layer varies from zero at the wall to  $M_e$  at the outer edge. In the expansion, the streamlines within the

subsonic portion of the boundary layer converge, whereas those in the supersonic portion diverge. The streamlines then undergo various turning angles  $\nu$ , and certainly, assuming one turning angle for the entire profile based on  $M_e$  [Eq. (4.23) of Ref. 9] and, in particular, when the velocity profile at separation is far from being "full" (not well-developed turbulent flow), as in the case of transition just upstream of the separation corner, leads to a large error in the predicted base pressure. However, even when the profile is "full," two distinct turning angles, characteristics of the supersonic and subsonic portions of the profile, must be introduced into the base-pressure solution. For example, the formation of lip shocks at the corner<sup>10,16,17</sup> may be caused by a non-smooth transition between the subsonic and supersonic portions of the boundary layer. Of course, these shocks may also be caused by an interaction between the fluid circulating in the "dead-air region" near the base and the main free-stream.

Therefore, within the range of validity of the assumed model for flow over a steep, lower supersonic range, we would expect the theory of Nash to deviate widely from the experiment at low values of  $\theta/t$ , and, in particular, as the boundary-layer thickness approaches zero, Eq. (1) ceases to be valid, and the limiting base pressure is grossly underestimated. However, the theory tends to agree more closely with the experiment as  $\theta/t$  increases, the agreement improving with decreasing Mach number (Figs. 3 and 4 of Ref. 9).

In spite of the preceding difficulties, the theory presents the first attempt to obtain an expression for the base pressure which takes quantitatively into account the fact that reattachment takes place before the pressure rises to its maximum value by including in the formulation of the base-pressure problem an additional parameter  $N$ , the ratio of the pressure rise to reattachment to the difference between free-stream static pressure and the base pressure, whose variation with Mach number is determined empirically. Furthermore, the theory is certainly an improvement over earlier theories<sup>12,13</sup> in predicting the variation of  $P_b/P_1$  with the momentum thickness at separation.

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## Production of Supersonic Flows with Honeycomb-Supported Screens

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### Introduction

IN the course of a special shock-tube development program,<sup>§</sup> the performance characteristics of honeycomb-supported screens for the production of supersonic flows was investigated using air and argon. Screens may be considered a variant form of the multinozzle that has been studied by several authors.<sup>1,2</sup> The principal advantages of the screen are its mechanical simplicity, the possibility of producing disturbance-free flows, and, relative to the nozzle, the very short flow-stabilization time for large-diameter ducts. The last feature is especially important for blow-down applications, as in the authors' case.

The only previous work, that of Gould,<sup>3</sup> shows the existence of substantial radial flow components emerging from the screen, as is evidenced by the pronounced shock diamonds,

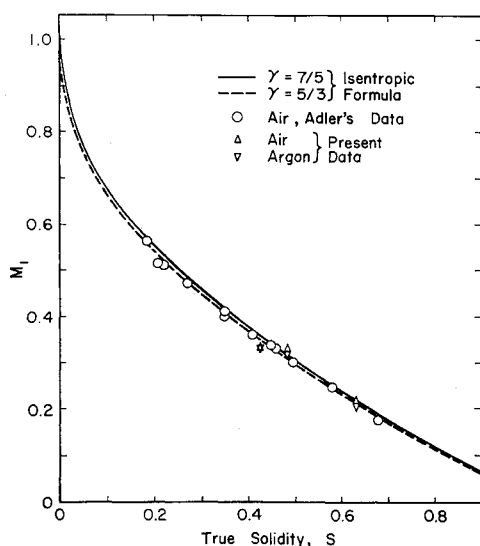


Fig. 1 Inlet Mach number as a function of true solidity.

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which caused the local Mach number to vary as much as 0.35 across the stream. The radial components can be attributed either to mechanical distortion of the screen or a convection through it of radial momentum produced by the flattening effect of the screen on the incoming velocity profile. Presumably because of strength limitations, Gould used quite coarse screens (9 to 20 mesh/in.) that gave a clearly visible fixed pattern of disturbances for several inches or more downstream. Unsteady fluctuations undoubtedly extended much further.

By means of a simple structural modification, resistance-welding a stainless-steel screen to the downstream side of a layer of stainless-steel honeycomb, the present authors have eliminated these difficulties. Two objectives are served thereby: 1) the honeycomb eliminates radial momentum coming into the screen; and 2) the closely spaced local support holds the screen flat and permits a much finer mesh screen to be used, resulting in a correspondingly finer scale and less persistent downstream disturbance pattern. These advantages are purchased with somewhat increased total pressure losses and a decreased maximum Mach number. The discussion falls naturally under the three major divisions.

### Upstream Flow State and Mass Flow Rate

Adler<sup>4</sup> and Cornell<sup>5</sup> state that the (choked flow) upstream Mach number  $M_1$  is determined by the screen solidity (= blocked area/total area) and the isentropic choked flow formula. However, at high solidities, Adler's  $M_1$  data are as much as 10% higher by this criterion. Since frictional effects cannot account for a change in this direction, it must be attributed to a somewhat increased "true" flow area relative to that which they computed. If a "wire-cloth" screen (i.e., both warp and fill wires have sine wave shapes) is made of wire of diameter  $d$  with a mesh  $m$ , the maximum and mean displacements from the center of one wire to the next are  $d$  and  $d/2^{1/2}$ , respectively, so that the aperture area is approximately  $[(1/m^2 + 1/2d^2)^{1/2} - d]^2$  instead of  $[1/m - d]^2$ . With the "bolting-grade" screen in which one family of wires is straight, the maximum and mean normal displacements of the curved wires are  $2d$  and  $2^{1/2}d$ , respectively, giving an aperture area of  $[(1/m^2 + 2d^2)^{1/2} - d][1/m - d]$ . The "normal" and "true" solidities of single screens are then

$$S_n = 1 - [1 - md]^2 \text{ "normal" solidity, any screen or square-cell honeycomb} \quad (1a)$$

$$S_{ts} = 1 - \{[1 + \frac{1}{2}(md)^2]^{1/2} - md\}^2 \text{ wire-cloth screen} \quad (1b)$$

$$S_{ts} = 1 - \{[1 + 2(md)^2]^{1/2} - md\}[1 - md] \text{ bolting-grade screen} \quad (1c)$$

By similar reasoning, and using the honeycomb  $m$  and  $d$  in Eq. (1a), the effective honeycomb solidity in the plane of the screen is estimated to be  $S_h = 0.6 S_n$ . The combined solidity and its relation to the area-ratio function  $A/A^*$  is

$$S = S_{ts} + S_h(1 - S_{ts}) = 1 - A^*/A \quad (2)$$

Table 1 Solidity data

	Mesh $m$ , in. <sup>-1</sup>	Wire diam $d$ , in.	Normal solidity $S_n$ , Eq. (1)	True solidity $S_{ts}$ , Eq. (1)	Com- bined solidity $S$ , Eq. (2)
Honeycomb	4	0.0060	0.048	0.029	...
Wire-cloth screen	40	0.0060	0.423	0.401	0.420
Bolting-grade screen	20	0.0162	0.543	0.475	0.491
Wire-cloth screen	60	0.0072	0.678	0.621	0.633